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To cite this article: P Markandeya Raju et al 2021 IOP Conf. Ser.: Mater. Sci. Eng. 1025 012005

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Cost optimization of a rectangular singly reinforced concrete beam by Generalized Reduced Gradient method

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Abstract. Cost Optimization of a singly reinforced rectangular beam using GRG method is presented in this paper. The objective function is chosen as the overall cost of the beam given by the sum of the products of unit rates of steel and concrete in rupees per unit volume (m^3) with their volumes. 28 combinations of simply supported beams of different spans subjected to different point loads and uniformly distributed loads are analyzed using EXCEL Solver. The constraints are adopted to satisfy various guidelines as per IS 456:2000 code of practice.

Keywords: Singly reinforced beam, Optimization, Excel solver, Cost-effective, Structural Design

1. Introduction

The word optimization denotes a process of making a design or a decision as effective as possible. It is generally done by minimizing or maximizing an objective function that satisfies various constraints. It plays a very important role in structural engineering by helping the engineers in designing a good, efficient, safe and economical structure. Various Optimization techniques developed by mathematicians have the potential to reduce the overall cost of engineering construction through the effective and relevant mathematical methodology. The main purpose of structural optimization is to find the best design out of several alternate designs without compromising on safety, serviceability and durability by effective utilization of available resources.

In this paper, Cost Optimized design of a singly reinforced rectangular beam using GRG (Generalized Reduced Gradient) method is presented. The objective function is chosen as the overall cost of the beam given by the sum of the products of unit rates of steel and concrete in rupees per unit volume (m^3) with their volumes. 480 combinations of simply supported beams of different spans subjected to different point loads and uniformly distributed loads are analyzed using EXCEL Solver [1] where the data, Objective function, other formulae for calculation and constraints are written in order. The different combinations are solved and finally, the most optimum combination of design structure is taken. The optimization is done for the singly reinforced beam. The constraints are adopted to satisfy various clauses of IS 456:2000 [2]. The rates of concrete and steel are taken in Indian currency i.e. Rupees per unit volume as per SSR

2. Literature review

Many researchers have been working in this area of cost optimization of RC beams. Some of the works and corresponding findings are presented as follows.

Vishakha Tushiram Holambe et al [4] used to define a constrained optimization function for a per meter cost of singly reinforced rectangular concrete (SRRC) beam. 21 combinations of the span of beam and loading were analyzed using the SWARM Optimization approach. The unit cost of materials and unit cost for side and bottom formwork was considered to be a major cost, the result which was obtained were compared with the depth of the beam, area of steel required and cost of beam per meter for balanced sections. Stency Mariam Thomas et al [5] presented an optimal design of reinforced concrete beams, its main objective is to minimize the total cost of the beam. The optimization was done for different grades of concrete and steel to know the better grade of concrete and steel the design was done through MATLAB software. The problem was formulated as a non-linear constrained minimization it was solved through fmincon SOP algorithm. A model of optimal design of rectangular reinforced concrete section is presented by Barros et al [6] considered the stress-strain diagram which EC2-2001 and MC90, the expressions like economic bending moment, optimal was described in area of steel and optimal steel ratio between upper and lower steel was developed. He had concluded that in non-dimensional form the equations were nearly coincident for both singly and doubly reinforcement. Cost optimization is implemented in code was compared with other optimum models based on the ultimate design of ACI. Sonia et al [7] presented a particle swarm optimization (POS) to achieve optimal design of Reinforced Concrete (RC) beams. They have developed an algorithm to search for a minimum cost solution that has to satisfy Indian code requirements for R.C beams. The objective function consisted of the cost of concrete and rebars as prevalent at the place of construction. Many examples were presented to show the effectiveness of the formulation for achieving optimal design. Cost optimization of concrete structures is presented by Kamal C Sarma and Hojjat Adeli [8] they presented a series of paper on cost optimization of concrete structures which was published in archival journals. The concrete structures must involve at least 3 different materials that are concrete, steel and formwork. The structures which are included are beams, slabs, columns, frame structures, bridges, water tanks, folded plates, shear walls, pipes and a tensile member. A review of reliabilitybased cost optimization is also included in this paper. They have concluded that there is a need to perform research on cost optimization of realistic 3-dimensional structures, especially large structures with hundreds of members where optimization can result in a substantial saving. Kaveh and Ahangaran [9] presented a paper on discrete cost optimization of composite floor system using social harmony research model. They have included the costs of concrete, steel beam and shear studs in the cost function. The design is based on AISC load and resistance factor design specification and plastic design concept. They have considered 6 decision variables for the objective function. They have presented a model which results in significant cost saving. A parametric study is performed which is used to investigate the effects of beam spans and loading. Optimal seismic design of the reinforced concrete shear wall-frame structure is presented by Ali Kaveh and Pooya Zakian [10] they have performed the seismic design of reinforced concrete (RC) dual system as an optimization problem for which the charged system search algorithm is utilized as an optimize. The databases were created on ACI seismic criteria for beams, columns and shear walls. Ordinary design constraints and effective seismic design constraints were taken. According to the result, the proposed methodology can be considered as a suitable practical approach for optimal seismic design of reinforced concrete shear wall-frame structures. Shahzad Umar et al [11] presented a paper on cost optimization of RC godown. The main aim of this paper is to achieve optimal design of reinforced concrete structures, optimal sizing and reinforcing for beam and column member in multi-bay and multi-storey structures which results in cost saving over typical practice design. Siddhant Lakkad and Panchal [12] presented a paper on cost optimization of reinforced concrete structure elements, the main aim of this paper is to achieve the optimal and ideal design of the reinforced concrete structure, reducing size and reinforcement for beam member and column member in multi-bay and multi-storey structures. Which is used in several software likes STAAD PRO, ETABS, STRUDS. Programming of the design of structural elements

beam and column has been done using the MATLAB program. This optimization task reduces 24.76% of the total approximate cost in beams and 13.79% in columns.

From the above review literature, it can be observed that each author has considered the objective function differently. Some have considered material quantity and some have considered the overall cost. Further, most of the authors missed some of the key constraints that define the performance of the RC beam. Hence there is a need to comprehensively study the cost optimization of RC beam considering all possible constraints.

3. Methodology

The objective of this paper is to optimize the cost of a singly reinforced rectangular beam. The following step-by-step methodology was followed.

Step 1: 480 combinations of simply supported beams (SSB) of different spans subjected to different point loads were considered for the study. The combinations are obtained by varying the following parameters.

Span (L)	: 1 m, 2 m,4 m and 8 m
Grade of Concrete	: M20, M25, M30, M35 and M40.
Grade of Steel	: Fe 250, Fe 415 and Fe 500.
Central Point Load (kN)	: 10, 20, 40, 80
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Uniformly distributed load (kN/m) : 3, 7, 10, 20

Step 2: An EXCEL [1] sheet to analyze and determine the optimized design of the beam for distributed and point loads are developed. GRG Method was adopted. The cost of different grades of steel and concrete are taken from SSR (2018-19) [03]. By changing the grade of concrete, the grade of steel (thereby their costs), Ultimate moment of resistance (Mu), Ultimate shear force (Vu), different objective functional values are obtained for different spans.

Step 3: From this value, the overall cost and percentage of steel are calculated for various point loads and uniformly distributed loads. While the effective span 'L' is changeable, the optimum breadth 'B' and 'depth' d are given by the EXCEL Solver.

Step 4: Graphs are drawn for different spans of the beam based on overall cost and percentage of steel combinations and analyzed.

The objective function which we have used for this singly reinforced beam is the total cost of the beam that includes the total cost of steel and concrete including fabrication and labour cost.

The expression is given by:

 $f(\mathbf{x}) = \mathbf{Cr} \times (\mathbf{L} \times \mathbf{B} \times \mathbf{D} - \mathbf{Vs}) + \mathbf{Cs} \times \mathbf{Vs} \times \rho$

f(x) = (Rate of concrete in rupees per unit volume (Rs. /m³)) × (Volume of concrete (m³)) + (Rate of steel (Rs. /kg.) × (Volume of steel in m³ × Density of steel in kg. /m³)

Where,

 $Vs = Volume of Steel in m^3$

Cr= Cost of concrete (Rs. $/m^3$)

Details required

Cs= Cost of steel (Rs. /kg.)

Cost of steel (Cs): 36,42,41.6 (Rs/kg) for different grades of steel

L= Length of the beam in m

B = Breadth of the beam in m

D= Overall depth of the beam in m

 ρ = Density of steel = 7700 kg/m³

Other notation/data used for derivation of objective function and determination of constraints are as follows:

Mu = Ultimate (factored) bending moment N/mm (due to dead and live load)

Vu = Ultimate (factored) vertical shear force in N (due to dead and live load)

fck = Compressive Strength of concrete = 20 MPa, 25 MPa, 30 MPa, 35 MPa, 40 MPa

For grades of Concrete: M20, M25, M30, M35, M40 respectively

International Conference on Sustainable Systems and Structures (ICSSS 2019) **IOP** Publishing IOP Conf. Series: Materials Science and Engineering 1025 (2021) 012005 doi:10.1088/1757-899X/1025/1/012005 $fy = Grade of Steel = 250 N/mm^2$, 415 N/mm², 500 N/mm² For grades of steel: Fe250, Fe 415, Fe 500 respectively. C = Force of compression = $0.36 \times fck \times B \times Xu$ T = Force of tension = $0.87 \times fy \times Ast$ Here. Ast = Area of steel in tension zone Moment of Resistance = $MR = 0.87 \times fy \times Ast \times (d - 0.42 \times Xu)$ (Clause 38.1 of IS 456 2000) Mu, lim = Ultimate moment of resistance of the mid-span = $k \times fck \times b \times d^2$ Where, k = Moment of resistance constant depending on the grade of steel) k = 0.148 for Fe 250, 0.138 for Fe 415 and 0.133 for Fe 500 $Xu = Depth of neutral axis of the section = \frac{0.87 \times fy \times Ast}{0.36 \times fck \times b}$ (Clause 38.1 of IS 456 2000) $\frac{Xu, lim}{d} = 0.53 \text{ for Fe } 250, 0.48 \text{ for Fe } 415 \text{ and } 0.46 \text{ for Fe } 500$ (Clause 38.1 of IS 456 2000) Initial values of cross-section and longitudinal dimensions L= 2000mm B=200mm (changeable) D=341mm (changeable) d'= Clear cover= 25mm (minimum clear cover if bar diameter less than or equal to 25mm) (Clause 26.4 of IS 456 2000) ds = Diameter of stirrups (8 mm, 10 mm, 12 mm)dts = Diameter of largest bar of tensile steel d = effective depth = D - (effective cover) = D - (d' + ds + (dts/2))Area of Longitudinal steel 2 numbers of 12 mm diameter bars are provided as anchor bars in compression zone. Asc= Area of steel in compression= $(2 \times \frac{\pi}{4} \times 12^2) = 226.19 \ mm^2$ Ast = Area of steel tension (limited to range within 226.28 mm^2 and 1473.21 mm^2) The area of steel tension (limited to range within 226.28 mm² and 1473.21 mm²) for different bar combinations adopted in this study is presented in Table 1.

Table 1. Area of l	iension sieei	<i>mm</i>) <i>j</i> 0.	i various bar	<u>combination</u> .
Area of tension	Bar	No.	Bar	No. of
	diameter	of	diameter	
steel (mm ²)	(mm)	bars	(mm)	bars
0226.28	12	2	00	0
0339.42	12	3	00	0
0402.28	16	2	00	0
0515.42	16	2	12	1
0603.42	16	3	00	0
0628.57	20	2	00	0
0741.71	20	2	12	1
0829.71	20	2	16	1
0942.85	20	3	00	0
0982.14	25	2	00	0
1095.28	25	2	12	1
1183.28	25	2	16	1
1296.42	25	2	20	1
1473.21	25	3	00	0

(Clause 40.4 of IS 456 2000)

(Clause 40.4 of IS 456 2000)

Area of lateral reinforcement

Sv=Spacing of 2 legged vertical stirrups of diameter 'ds' in mm (without bent up bars)

$$S_v$$

$$=\frac{(0.87 \times fy \times Asv \times d)}{Vus}$$

Where

Asv = Area of 'ds mm' diametre 2 legged vertical stirrups = $\frac{2 \times \pi \times ds^2}{4}$

Strength of the shear reinforcement = $Vus = Vu - Tc \times B \times D$ Nc = Values are taken from Table 19 of IS456:2000 based grade of concrete and Ast provided n = Number of Stirrups = $\frac{\text{Length over which stirrups are provided}}{2000 \text{ stirrups are provided}} + 1$

Spacing of stirrups spacing of stirrups Length over which stirrups are provided = $L - 2 \times d^2$

Hence,

$$n = \frac{L - 2d'}{S_v} + 1$$

Length of stirrup = perimeter of stirrup + length of hooks Perimeter of stirrup = $2 \times \{(b - 2d') + (D - 2d')\}$ Length of hooks = $2 \times (24 \times ds)$ (*Clause* 26.2.2.1 *of IS* 456 2000) Length of stirrup = $2 \times \{(b - 2d') + (D - 2d') + (24 \times ds)\}$ Asts = Area of stirrups = $\frac{\pi \times ds^2}{4}$

- Vu

$$B \times D$$

Cost of different grades of concrete

Table 2 presents the Nominal concrete mix ratios (by volume and weight) that are considered for the estimation of the cost of different grades of concrete. The rate of Cement, Fine aggregate, Coarse aggregate, fabrication and labour cost were adopted as per SSR [2].

Table 2. Nominal mix ratios for different grades of concrete (by volume and weight)

M5	1	5	10	Parts
IVI3	146	751	1502	kg
M7.5	1	4	8	Parts
IVI /	180	740	1480	kg
M10	1	3	6	Parts
WI10	235	721	1443	kg
M15	1	2	4	Parts
NI13	335	688	1376	kg
M20	1	1.5	3	Parts
M120	427	657	1315	kg
M25	1	1	2	Parts
W123	589	603	1207	kg
M30	1	0.75	1.5	Parts
M30	726	557	1115	kg
M25	1	0.5	1	Parts
M35	946	484	969	kg
M40	1	0.25	0.5	Parts
10140	1357	347	695	kg

Table 3 presents the sample calculation for gross cost of M30 grade concrete and Fe 415 steel including centering cost, contractors profit and labour cost.

Note: Supply and placing of the Design Mix Concrete corresponding to IS 456 using Weigh Batcher / Mixer with 20mm size graded machine crushed hard granite metal (coarse aggregate) from approved quarry including cost and conveyance of all materials like cement, fine aggregate (sand) coarse aggregate, water etc., to site and including Seigniorage charges, sales & other taxes on all materials including all operational, incidental and labour charges such as weigh batching, machine mixing, laying concrete, curing etc., complete but excluding cost of steel and its fabrication charges for finished item of work (APSS No. 402)with minimum cement content as per IS code from standard suppliers approved by the department including pumping, centering, shuttering, laying concrete, vibrating, curing etc. complete but excluding cost of steel and its fabrication charges for finished item of work.

Table 3. Samp	le calculation	for cost of M30	grade concrete an	d Fe 415 grade steel
I able 5. Sump	ic cuichanon	<i>joi</i> cosi oj m 50	grade concrete an	

FF Beams plain (not Hilly) areas					
PLAIN CONCRETE BASIC COST			5037.67		
per 1 cum (M30 grade concrete) up to 150m			5057.07		
Centering charges for structural members (Beams)	cum	1.000	2868.00		
as per SSR 2018-19 (in Rs. per cubic meters)	cum	1.000	2000.00		
Total			7905.67		
Contractor profit @13.615%			1076.36		
MA @ 40% on labor			1117.66		
(labour compound on centering @1473/-)			1117.00		
Grand Total			10099.8		
Fe415 grade steel	MT	1.000	42500		

4. Derivation for Objective function

The Volume of steel = Volume of Longitudinal steel + Volume of Lateral steel

The volume of Longitudinal steel = product of Sum of the areas of steel in tension and compression zone and length of the bars

Here, Length of bars = Overall span – 2 times clear cover = $L - 2 \times d^2$ Hence, the Volume of Longitudinal steel = $(Ast + Asc) \times (L - 2d^2)$ The volume of Lateral steel (stirrups) = number of stirrups × area of stirrup × Length of the stirrup $V_{n} = \left[\frac{L - 2d^2}{2} + 1\right] \times \left[\frac{\pi \times ds^2}{2}\right] \times \left[2 \times \left\{(h - 2d^2) + (D - 2d^2) + (24 \times ds)\right\}\right]$

$$V_s = \left[\frac{1}{S_v} + 1 \right] \times \left[\frac{1}{4} \right] \times \left[2 \times \left\{ (b - 2d^{\prime}) + (D - 2d^{\prime}) + (24 \times ds) \right\} \right]$$

The volume of concrete = Total Volume of Beam – Volume of Steel Vc = V – Vs = L×B×D- Vs

5. Constraints

1) Neutral axis constraint: $Xu \le Xu,max$

2) Moment constraint: $Mu \le Mu$, lim

3) Deflection constraint:

$$d \le \sqrt{\frac{Mu}{k \times fck \times b}}$$
 (Clause 38.1 of IS 456 2000)

4) For spans up to 10m constraint:

$$\frac{l}{d} \le 20$$
 (Clause 23.2.1 of IS 456 2000)

5) For spans > 10m constraint:

$$\frac{l^2}{d} \le 200 \text{ if } l < 10m, \text{else } \frac{l}{d} \le 20 \text{ (Clause 23.2.1 of IS 456 2000)}$$

6) The maximum area of steel constraint:

$$\frac{Ast}{B \times d} \le 0.04 \ (Clause \ 26.5.1.1(b) \ of \ IS \ 456 \ 2000)$$

7) The minimum area of steel constraint:

$$\frac{Ast}{B \times d} \ge \frac{0.85}{fy}$$
(Clause 26.5.1.1(a) of IS 456 2000)

8) Constraint to restrict the beam to be under-reinforced (to be designed as singly reinforced):

$$\frac{\frac{Xu}{d}}{\frac{Xu, lim}{d}} - 1 < 0 \ (Clause \ 38.1 \ of \ IS \ 456 \ 2000)$$

9) Recommended minimum beam depth to width ratio:

$$b < \frac{a}{1}$$

10) Recommended maximum beam depth to width ratio:

$$b > \frac{a}{3}$$

11) Minimum width constraint (assumed): $B \ge 200$

- 12) Maximum width constraint (assumed): $B \le 600$
- 13) Minimum depth constraint (assumed): $d \ge 200$
- 14) Maximum width constraint (assumed): $d \le 600$
- 15) Minimum Ast constraint (rounded off to nearest multiple of 100): Ast ≥ 200
- 16) Maximum Ast constraint (rounded off to nearest multiple of 100): Ast < 1500
- 17) The constraint for ensuring positive value for Volume of steel: The volume of steel > 0

18) The constraint for ensuring that Moment of resistance of the mid-span section is greater than the Ultimate (factored) bending moment: MR > Mu

The objective function and the constraints are programmed into the EXCEL Solver. Table 4 presents the EXCEL result sheet on running the EXCEL Solver for the following sample data.

Grade of Concrete	: M40	Ultimate Moment	: 40 kN_m
Grade of steel	: Fe415	Ultimate Shear force	: 80 kN
Effective span	: 2000 mm		

Table 4. Excel sneet calculations						
Objective function	2551.9			d	300	mm
CR (Cost of Concrete)	10099.68	Rs./m ³		Vus (Cl. 40.4)	1	
Grade of concrete	M40			С	143319.25	Ν
Grade of steel	Fe415			Т	143319.25	Ν
k	0.138			Xu (Cl. 38.1)	49.76	mm
fy	415	N/mm ²		Mu (Cl. 38.1)	40000300.02	N_mm
Cs (Cost of Steel)	42	Rs./kg		Mu,lim (<i>Cl. 38.1</i>)	99360000	N_mm
L (Effective Span)	2000	mm		Xu,lim./d (<i>Cl. 38.1</i>)	0.48	
Mu (Ultimate Moment)	40	kN_m		Min.Sv	225	mm
Vu (Ultimate Shear force)	80	kN		(L^2/d)-200	13133.33	
fck (Char. Comp. Strength of Concrete)	40	N/mm ²		(L/d)-20	-13.33	

Table 4. Excel sheet calculations

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doi:10.1088/1757-899X/1025/1/012005
doi:10.1000/1/2/ 0//1023/1/012003

В	200	mm	C1	-94.24	<=0
D	341	mm	C2	-59359699.98	<=0
Ast	396.95	mm ²	C3	299.99	>=0
Bar Combination.			C4	-14.13	<=0
d'	25	mm	C5	-13.33	<=0
w (Density of Steel)	7700	kg/m ³	C6	-0.034	<=0
Asc	226.19	mm ²	C7	0.005	>=0
Vol. of Steel	3748390.22	mm ³	C8	-0.654	<0
100Ast/BD	0.498	%	C9	0	<0
Tv (Cl. 40.4)	1.17	N/mm ²	C10	100	>0
Tc (Cl. 40.1)	0.568	N/mm ²	C11	0	>=0
Sv (<i>Cl.</i> 40.4)	225	N/mm ²	C12	-400	<=0
ds	8	mm	C13	100	>=0
Vc	132651609.8	Ν	C14	-300	<=0
dc	25	mm	C15	196.95	>=0
Vsc	45909.75	Ν	C16	-1103.05	<0
Ν	10		C17	3748390.22	>0
dts	16	mm	C18	300.02	>0
Xu,lim	144	mm			

6. Results and discussion

The results presented graphically in 4 parts. Although the calculations are performed for all grades of concrete ranging from M20 to M40 (in intervals of 5, the results are presented here only for M20 and M40 to minimize the length of the manuscript.

Part 1 presents the variation of the percentage of steel with span for different point loads. Each graph presents results for a given grade of steel and concrete.

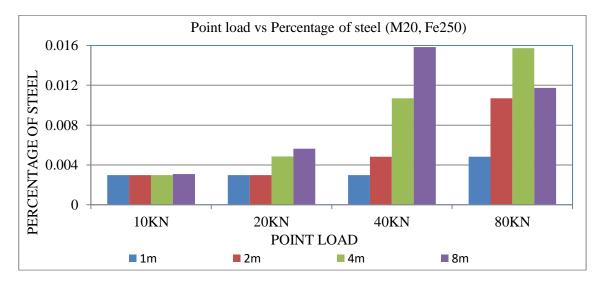


Figure 1. Variation of the percentage of steel with span for different loads (for Fe250, M20)

International Conference on Sustainable Systems and Str	IOP Publishing	
IOP Conf. Series: Materials Science and Engineering	1025 (2021) 012005	doi:10.1088/1757-899X/1025/1/012005

From figure. 1, it can be observed that for a span 1 m, at 10 KN, the percentage of steel is 0.002969. It slightly decreases to 0.002965 at 20kN and remains constant till 40 kN. Then increases to 0.004829 at 80 kN. For a span 2 m, at 10 kN, the percentage of steel is 0.002965 and remains constant till 20 kN. Then it increases to 0.004829 at 40kN and gradually increases to 0.010699 at 80 kN. For a span 4 m, at 10 kN, the percentage of steel is 0.002982. It increases to 0.004847 at 20 kN. Then it increases to 0.010699 at 40kN and gradually increases to 0.015736 at 80kN. For a span 8 m, at 10 kN, the percentage of steel is 0.003085. Then increases to 0.00564 at 20kN and it increases to 0.015852 at 40kN, later it decreases to 0.011728 at 80 kN.

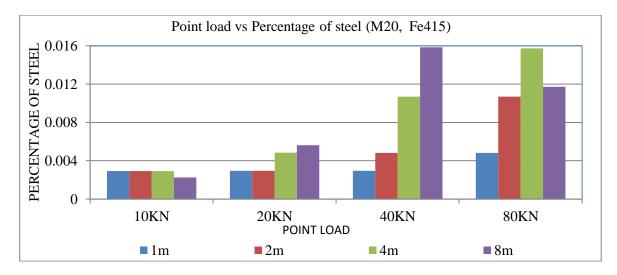
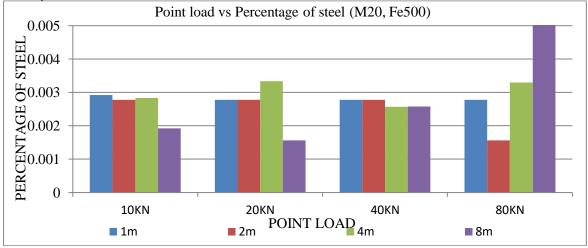
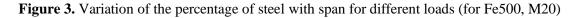


Figure 2. Variation of the percentage of steel with span for different loads (for Fe415, M20)

From figure 2, it can be observed that for a span 1 m, at 10 kN, the percentage of steel is 0.002923. It slightly increases to 0.002964 at 20kN and remains constant till 40 kN. Then it increases to 0.004829 at 80kN. For a span 2 m, at 10 kN, the percentage of steel is 0.002923. It slightly increases to 0.002964 at 20 kN. Then it increases to 0.004828 at 40kN and 0.010699 at 80kN. For a span 4 m, at 10 kN, the percentage of steel is 0.002932. It increases to 0.004846 at 20 kN. Then it increases to 0.015735 at 80kN. For a span 8 m, at 10 kN, the percentage of steel is 0.002247. Then it increases to 0.00563 at 20kN and increases to 0.015851 at 40 kN. Then gradually decreases to 0.011727 at 80 kN.





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From figure 3, it can be observed that for a span 1m, at 10 kN, the percentage of steel is 0.002923. It slightly decreases to 0.002778 at 20kN and remains constant till 80 kN. For a span 2m, at 10 kN, the percentage of steel is 0.002777 and remains constant till 40kN. Then it decreases to 0.00156 at 80 kN. For a span 4m, at 10 kN, the percentage of steel is 0.002834. Then it increases to 0.003336 at 20 kN. Later it decreases to 0.002568 at 40 kN. Then it gradually increases to 0.003294 at 80kN. For a span 8m, at 10 kN, the percentage of steel is 0.001923. Then decreases to 0.00156 at 20kN, increases to 0.002576 at 40kN and further increases to 0.005021 at 80kN.

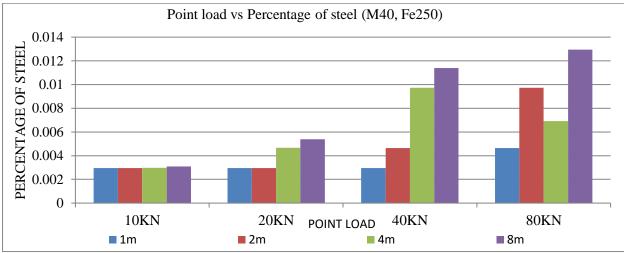
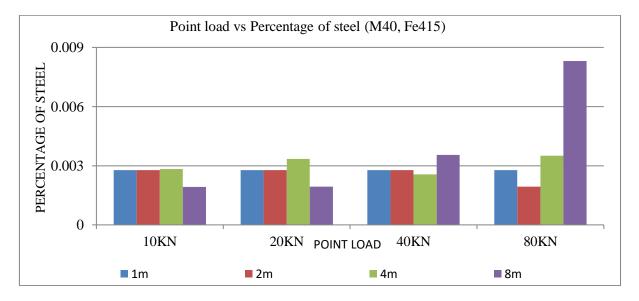
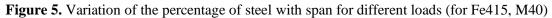


Figure 4. Variation of the percentage of steel with span for different loads (for Fe250, M40)

From figure 4, it can be observed that for a span 1m, at 10 kN, the percentage of steel is 0.002964 and remains constant till 40 kN. Then increases to 0.004644 at 80 kN. For a span 2m, at 10 kN, the percentage of steel is 0.002964 and remains constant till 20kN. Then it increases to 0.004643 at 40kN and 0.009731 at 80 kN. For a span 4 m, at 10 kN, the percentage of steel is 0.002982. It increases to 0.004674 at 20 kN. Then it increases to 0.009731 at 40 kN and decreases to 0.0069227 at 80kN. For a span 8 m, at 10 kN, the percentage of steels 0.003085. Then increases to 0.005378 at 20 kN and increases to 0.0011388 at 40 kN. Then increases to 0.012953 at 80 kN.





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IOP Conf. Series: Materials Science and Engineering	1025 (2021) 012005	doi:10.1088/1757-899X/1025/1/012005

From figure 5, it can be observed that for a span 1 m, at 10 kN, the percentage of steel is 0.002777 and remains constant till 80 kN. For a span 2 m, at 10 kN, the percentage of steel is 0.002777 and remains constant till 40kN. Then it decreases to 0.001940 at 80kN. For a span 4 m, at 10 kN, the percentage of steel is 0.002834. It increases to 0.003340 at 20 kN. Then it decreases to 0.002554 at 40 kN and increases to 0.003505 at 80kN. For a span 8 m, at 10 kN, the percentage of steels 0.001940at 20 kN and increases to 0.003542 at 40 kN. Then increases to 0.008306 at 80 kN.

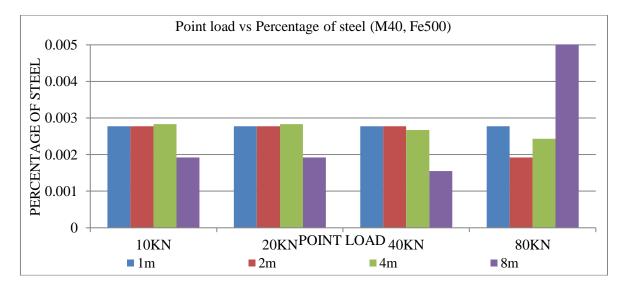


Figure 6. Variation of the percentage of steel with span for different loads (for Fe500, M40)

From figure 6, it can be observed that for a span 1 m, at 10 kN, the percentage of steel is 0.002777 and remains constant till 80kN. For a span 2 m, at 10 kN, the percentage of steel is 0.002777 and remains constant till 40 kN. Then it decreases to 0.001923 at 80kN. For a span 4 m, at 10 kN, the percentage of steel is 0.002834. It increases to 0.002835 at 20 kN. Then it decreases to 0.002674 at 40 kN and decreases 0.002432 at 80kN. For a span 8 m, at 10 kN, the percentage of steels 0.001923 and remains constant till 20kN and decreases to 0.001549 at 40 kN. Then increases to 0.005021 at 80 kN.

Part 1 present's variation of the optimum percentage of steel for a given span and point load. From the results, it can be concluded that for lower grades of steel, the magnitude of the optimum percentage of steel is almost same for all spans at lesser loads while it increases with span for higher loads. However, for higher grades of steel, the variation of the percentage of steel with span and point load is not very significant.

Part 2 presents the variation of the percentage of steel with span for different uniformly distributed loads. Each graph presents results for a given grade of steel and concrete.

From figure 7, it can be observed that for a span 1 m, at 3 kN/m, the percentage of steel is 0.002965 and remains constant till 20kN/for a span 2 m, at 3 kN/m, the percentage of steel is 0.002965 and remains constant till 20kN/m. For a span 4 m, at 3 kN/m, the percentage of steel is 0.002982. It increases to 0.003328 at 7 kN/m. Then it increases to 0.004847 at 10 kN/m and 0.010699 at 20kN/for a span 8 m, at 3 kN/m, the percentage of steel is 0.002982. Then increases to 0.008186 at 7 kN/m and increases to 0.012681 at 10 kN/m. Then decreases to 0.011802 at 20 kN/m.

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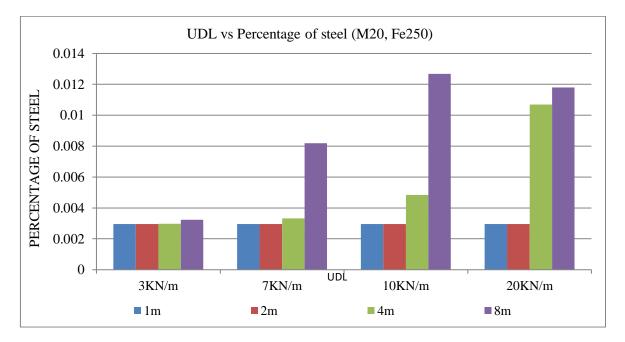


Figure 7. Variation of the percentage of steel with span for different loads (for Fe250, M20)

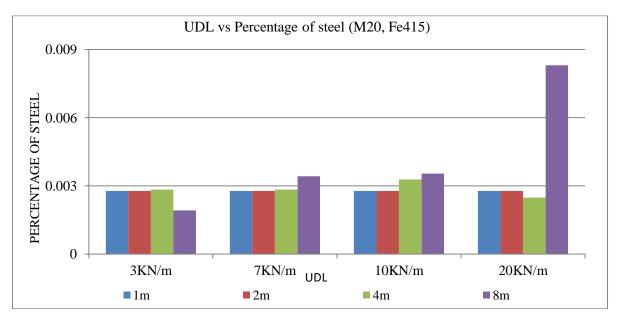


Figure 8. Variation of the percentage of steel with span for different loads (for Fe415, M20)

From figure 8, it can be observed that for a span 1 m, at 3 kN/m, the percentage of steel is 0.002777 and remains constant till 20 kN/for a span 2 m, at 3 kN/m, the percentage of steel is 0.002777 and remains constant till 20 kN/m. For a span 4 m, at 3 kN/m, the percentage of steel is 0.002834 and remains constant till 7kN/m. Then it increases to 0.003277 at 10 kN/m and decreases to 0.002482 at 20kN/for a span 8 m, at 3 kN/m, the percentage of steel is 0.002482 at 20kN/for a span 8 m, at 3 kN/m. Then increases to 0.003277 at 10 kN/m and increases to 0.003542 at 10 kN/m. Then increases to 0.008306 at 20 kN/m.

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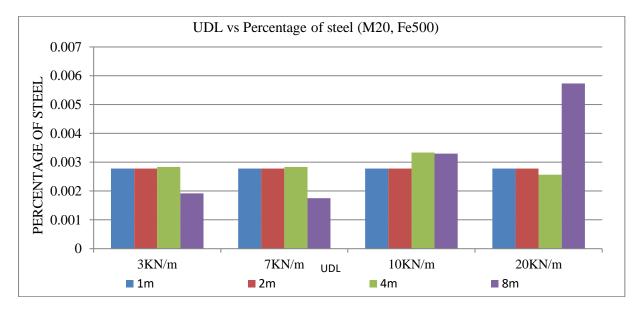


Figure 9. Variation of the percentage of steel with span for different loads (for Fe500, M20)

From figure 9, it can be observed that for a span 1 m, at 3 kN/m, the percentage of steel is 0.002778 and remains constant till 20 kN/for a span 2 m, at 3 kN/m, it is 0.002778 and remains constant till 20kN/m. For a span 4 m, at 3 kN/m, it is 0.002835 and remains constant till 7 kN/m. Then it increases to 0.003336 at 10 kN/m and decreases to 0.002568 at 20kN/for a span 8 m, at 3 kN/m, the percentage of steel is 0.001749 at 7kN/m and increases to 0.003302 at 10 kN/m and then increases to 0.005731 at 20 kN/m.

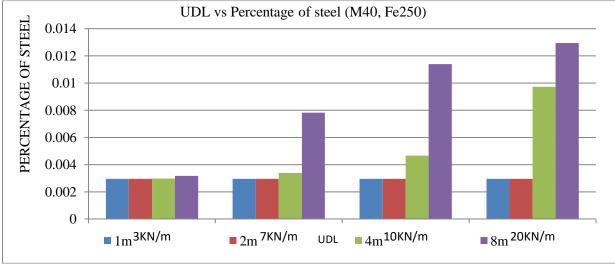


Figure 10. Variation of the percentage of steel with span for different loads (for Fe250, M40)

From figure 10, it can be observed that for a span 1 m, at 3 kN/m, the percentage of steel is 0.00296 and remains constant till20 kN/for a span 2 m, at 3 kN/m, the percentage of steel is 0.00296 and remains constant till 20kN/m. For a span 4 m, at 3 kN/m, the percentage of steel is 0.00298. It increases to 0.00338 at 7 kN/m. Then it increases to 0.00467 at 10 kN/m and 0.00973 at 20kN/for a span 8 m, at 3 kN/m, the percentage of steel is 0.001139 at 10 kN/m. Then increases to 0.01295 at 20 kN/m.

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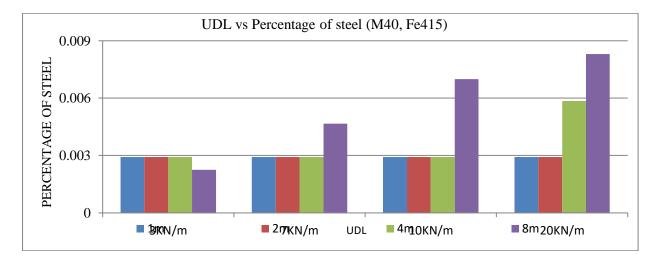


Figure 11. Variation of the percentage of steel with span for different loads (for Fe415, M40)

From figure 11, it can be observed that for a span 1 m, at 3 kN/m, the percentage of steel is 0.00292 and remains constant till 10 kN/m. Then it increases to 0.002923 at 20kN/for a span 2 m, at 3 kN/m, the percentage of steel is 0.00292 and remains constant till 10 kN/m. Then it increases to 0.002923 at 20kN/for a span 4 m, at 3 kN/m, the percentage of steel is 0.00293 and remains constant till 10 kN/m. Then it increases to 0.002923 at 20kN/for a span 4 m, at 3 kN/m, the percentage of steel is 0.00293 and remains constant till 10 kN/m. Then it increases to 0.005847 at 20kN/for a span 8 m, at 3 kN/m, the percentage of steel is 0.00295. Then increases to 0.00467 at 7 kN/m and increases to 0.00699 at 10 kN/m. Then increases to 0.008307 at 20 kN/m.

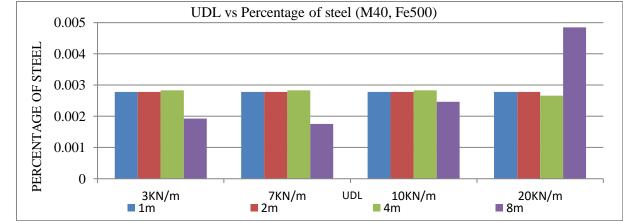
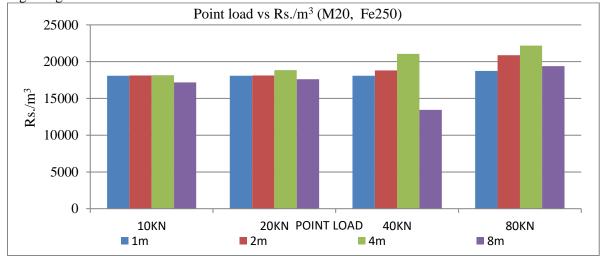


Figure 12. Variation of the percentage of steel with span for different loads (for Fe500, M40)

From figure 12, it can be observed that for a span 1 m, at 3 kN/m, the percentage of steel is 0.002777 and remains constant till 20 kN/for a span 2 m, at 3 kN/m, the percentage of steel is 0.002777 and remains constant till 20kN/m. For a span 4 m, at 3 kN/m, the percentage of steel is 0.002834 and remains constant till 7 kN/m. Then it increases to 0.002835 at 10 kN/m and 0.002659 at 20kN/for a span 8 m, at 3 kN/m, the percentage of steel is 0.001923. Then decreases to 0.001754 at7 kN/m and increases to 0.002468 at 10 kN/m. Then increases to 0.004849 at 20 kN/m.

Part 2 presents the variation of the optimum percentage of steel for a given span and uniformly distributed load. From the results, it can be concluded that for lower grades of steel, the magnitude of the optimum percentage of steel is almost same for all spans at lesser loads while it increases with span for higher loads. However, for higher grades of steel, the variation of the percentage of steel with span and point load is not very significant.



Part 3 presents the variation of $cost/m^3$ in Rs. with span for different loads. Each graph presents results for a given grade of steel and concrete.

Figure 13. Variation of cost/m³ (in Rs.) with span for different loads (for Fe250, M20)

From figure 13, it can be observed that for a span 1 m, at 10 kN, the Cost /m³ is Rs. 18069.35. It slightly decreases to 18067.76at 20 kN. Then it increases to 18069.35 at 40 kN and 18725.89 at 80 kN. For a span 2 m, at 10 kN, it is 18123.9 and remains constant till 20 kN. Then it increases to 18799.68 at 40 kN and 20984.46 at 80 kN. For a span 4 m, at 10 kN, it is 18144.98. It increases to 18843.45 at 20 kN. Then it increases to 21049.18 at 40 kN and 22180.54 at 80 kN. For a span 8 m, at 10 kN, it is 17161.72 and slightly decreases to 17161.71 at 20 kN and decreases to 13442.79 at 40 kN. Then increases to 19386.2 at 80 kN.

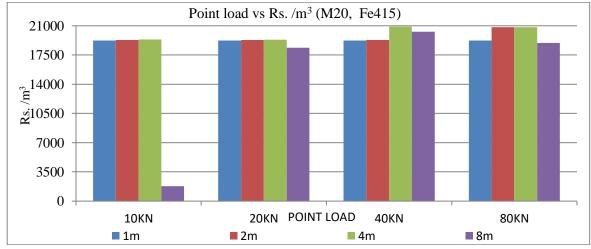


Figure 14. Variation of cost/m³ (in Rs.) with span for different loads (for Fe415, M20)

From figure 14, it can be observed that for a span 1 m, at 10 kN, the cost/m³ is 19230.23 and remains constant till 80 kN. For a span 2 m, at 10 kN, the cost/m³ is 19295.34 and remains constant till 40 kN and increases to 20827.05 at 80 kN. For a span 4 m, at 10 kN, the $cost/m^3$ is 19347.33. It decreases to 19332.12 at 20 kN. Then it increases to 20886.37 at 40 kN and decreases to 20825.93 at 80 kN. For a span 8 m, at 10 kN, the $cost/m^3$ is 1783.41. It increases to 18368.30 at 20 kN and increases to 20274.71 at 40 kN. Then decreases to 18948.83 at 80 kN.

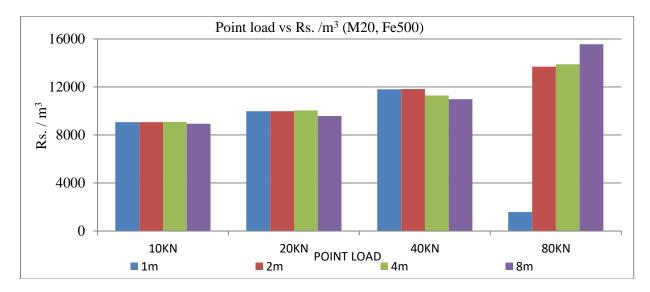


Figure 15. Variation of cost/m³ in Rs. with span for different loads (for Fe500, M20)

From figure 15, it can be observed that for a span 1 m, at 10 kN, the $cost/m^3$ is Rs. 9069.305. It increases to 9983.527 at 20 kN. Then it increases to 11812.48 at 40 kN and decreases to 1581.486 at 80 kN. For a span 2 m, at 10 kN, it is 9073.25. It increases to 9993.076 at 20 kN. Then it increases to 11832.72 at 40 kN and 13687.92 at 80 kN. For a span 4 m, at 10 kN, the $cost/m^3$ is 9083.983. It increases to 10051.48 at 20 kN. Then it increases to 11290.52 at 40 kN and 13884.09 at 80 kN. For a span 8 m, at 10 kN, it is 8946.585. It increases to 9579.107 at 20 kN and increases to 10984.52 at 40 kN. Then increases to 15550.20 at 80 kN.

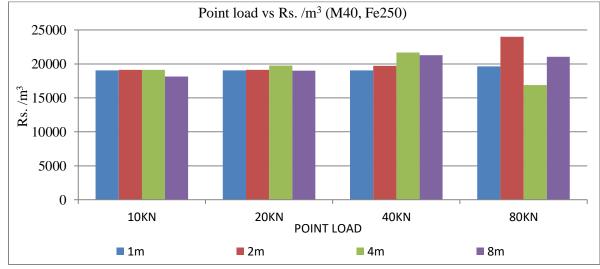


Figure 16. Variation of cost/m³ in Rs. with span for different loads (for Fe250, M40)

From figure 16, it can be observed that for a span 1 m, at 10 kN, the $cost/m^3$ is Rs. 19049. It increases to 19049.03 at 20 kN and remains constant till 40 kN. Then it increases to 19639.05 at 80 kN. For a span 2 m, at 10 kN, it is 19105.02 and remains constant till 20 kN. Then it increases to 19710.78 at 40 kN and 23989.38 at 80 kN. For a span 4 m, at 10 kN, the $cost/m^3$ is 19126.04. It increases to 19755.38 at 20 kN. Then it increases to 21660.96 at 40 kN and decreases to 16895.34 at 80 kN. For a span 8 m, at 10 kN, it is 18145.39. It increases to 19007.53 at 20 kN and increases to 21291.12 at 40 kN. Then decreases to 21044.44 at 80 kN.

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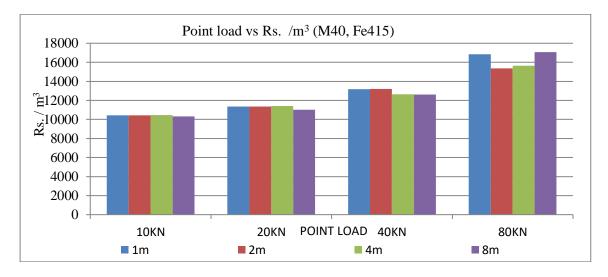


Figure 17. Variation of cost/m³ in Rs. with span for different loads (for Fe415, M40)

From Figure 17, it can be observed that for a span 1 m, at 10 kN, the cost/m³ is Rs. 10431.4. It increases to 11345.87 at 20 kN. Then it increases to 13174.81 at 40 kN and 16832.72 at 80 kN. For a span 2 m, at 10 kN, it is 10435.4. It increases to 11355.22 at 20 kN. Then it increases to 13194.86 at 40 kN and 15361.41 at 80 kN. For a span 4 m, at 10 kN, it is 10445.63. It increases to 11412.76 at 20 kN. Then it increases to 12655.97 at 40 kN and 15642.82 at 80 kN. For a span 8 m, at 10 kN, it is 10314.59. It increases to 11018.64 at 20 kN and increases to 12606.85 at 40 kN. Then increases to 17068.59 at 80 kN.

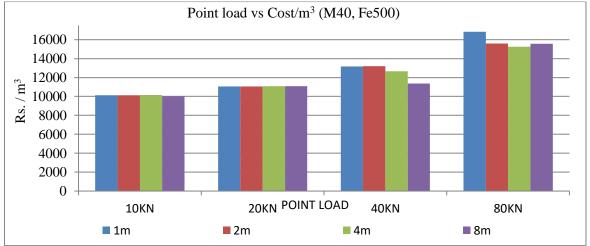


Figure 18. Variation of cost/m³ in Rs. with span for different loads (for Fe500, M40)

From figure 18, it can be observed that for a span 1 m, at 10 kN, the cost/m³ is Rs. 10132.19. It increases to 11046.66 at 20 kN. Then it increases to 13174.34 at 40 kN and 16832.23 at 80 kN. For a span 2 m, at 10 kN, it is 10136.23. It increases to 11056.05 at 20 kN. Then it increases to 13194.38 at 40 kN and 15599.32 at 80 kN. For a span 4 m, at 10 kN, the cost/m³ is 10146.57. It increases to 11080.14 at 20 kN. Then it increases to 12672.54 at 40 kN and 15256.35 at 80 kN. For a span 8 m, at 10 kN, it is10014.14. It increases to 11072.17 at 20 kN and increases to 11357.81 at 40 kN. Then increases to 15561.16 at 80 kN.

Part 3 presents the variation of $cost /m^3$ in Rs. for a given span and central point load. From the results, it can be concluded that for lower grades of steel and concrete, the $cost / m^3$ in Rs. is almost the

same for all spans and loads. At higher grades of concrete, it is constant for a given load and independent of the span. However, the cost $/m^3$ increase with span for higher grades of concrete.

Part 4 presents the variation of cost/m³ in Rs. with span for different uniformly distributed loads. Each graph presents results for a given grade of steel and concrete.

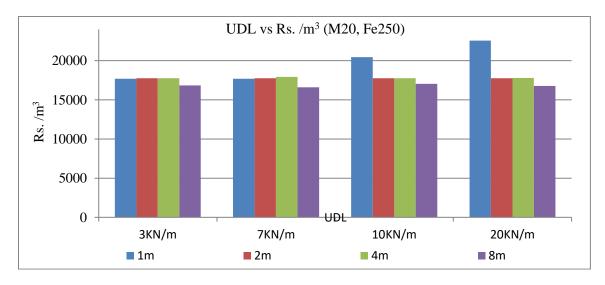


Figure 19. Variation of cost/m³ in Rs. with span for different loads (for Fe250, M20)

From figure 19, it can be observed that for a span 1 m, at 3 kN/m, the cost/m³ is Rs. 17688.61 and remains constant till 7 kN/m. Then it increases to 20449.46 at 10 kN/m and 22553.06 at 20 kN/m. For a span 2 m, at 3 kN/m, it is17744.81 and remains constant till 20kN/m. For a span 4 m, at 3 kN/m, it is 17769.62. It increases to 17910.08 at7 kN/m. Then it decreases to17756.03 at 10 kN/m and 17772.91 at 20kN/m. For a span 8 m, at 3 kN/m, it is 16832.19. Then decreases to 16601.21at 7kN/m and increases to 17047.64 at 10 kN/m. Then decreases to 16767.77 at 20 kN/m.

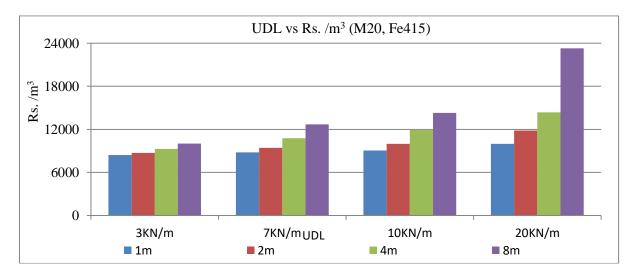


Figure 20. Variation of cost/m³ in Rs. with span for different loads (for Fe415, M20)

From figure 20, it can be observed that for a span 1 m, at 3 kN/m, the cost/m³ is Rs. 8429.20. It increases to 8795 at 7 kN/m. Then it increases to 9069.34 at 10 kN/m and 9983.81 at 20 kN/m. For a

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span 2 m, at 3 kN/m, it is 8705.60. It increases to 9441.46 at 7 kN/m. Then it increases to 9993.36 at 10 kN/m and 11833 at 20 kN/m. For a span 4 m, at 3 kN/m, it is 9270.96. It increases to 10764.70 at 7 kN/m. Then it increases to11948.50 at 10 kN/m and 14371.59 at 20kN/m. For a span 8 m, at 3 kN/m, the $cost/m^3$ is 10008.13. Then increases to 12683.64 at 7 kN/m and increases to 14290.88 at 10 kN/m. Then increases to 23270.42 at 20 kN/m.

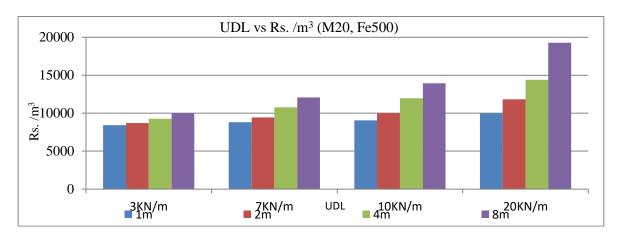


Figure 21. Variation of cost/m³ in Rs. with span for different loads (for Fe500, M20)

From figure 21, it can be observed that for a span 1 m, at 3 kN/m, the $cost/m^3$ is Rs. 8429.22. It increases to 8795.009 at 7 kN/m. Then it increases to 9069.351 at 10 kN/m and 9983.824 at 20 kN/m. For a span 2 m, at 3 kN/m, it is 8705.611. It increases to 9441.468 at 7 kN/m. Then it increases to 9993.362 at 10 kN/m and 11833.01 at 20 kN/m. For a span 4 m, at 3 kN/m, it is 9270.963. It increases to 10764.76 at 7 kN/m. Then it increases to 11956.91 at 10 kN/m and 14396.79 at 20kN/m. For a span 8 m, at 3 kN/m, it is 10008.14. Then increases to 12077.82 at 7 kN/m and increases to 13926.43 at 10 kN/m. Then increases to 19291.98 at 20 kN/m.

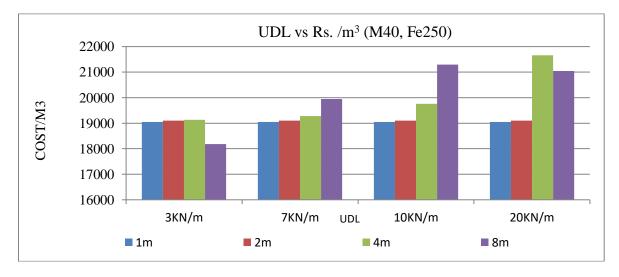
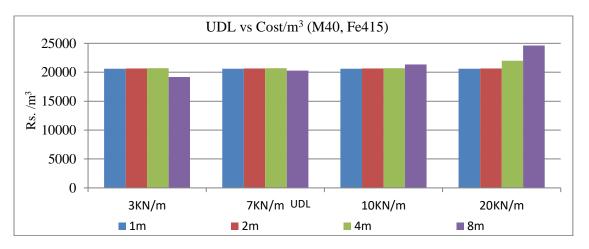


Figure 22. Variation of cost/m³ with span for different loads (for Fe250, M40)

From figure 22, it can be observed that for a span 1 m, at 3 kN/m, the cost/m³ is Rs. 19049.5 and remains constant till 20 kN/m. For a span 2 m, at 3 kN/m, the cost/m³ is Rs. 19105.5 and remains constant till20 kN/m. For a span 4 m, at 3 kN/m, the cost/m³ is 19130.2. It increases to 19275.1 at7 kN/m. Then it increases to 19755.9 at 10 kN/m and 21661.4 at 20kN/m. For a span 8 m, at 3 kN/m, the



cost/m³ is 18177. Then increases to 19950.6 at 7 kN/m and increases to 21291.6 at 10 kN/m. Then decreases to 21044.9 at 20 kN/m.

Figure 23. Variation of cost/m³ Rs. with span for different loads (for Fe415, M40)

From figure 23, it can be observed that for a span 1 m, at 3 kN/m, the cost/m³ is Rs. 20591.6 and remains constant till 10 kN/m. Then it decreases to 20591.56 at 20 kN/m. For a span 2 m, at 3 kN/m, the cost/m³ is 20656.5 and remains constant till 10 kN/m. Then it decreases to 20656.47 at 20kN/m. For a span 4 m, at 3 kN/m, the cost/m³ is 20708 and remains constant till 7 kN/m. Then it decreases to 20694.7 at 10 kN/m and increases to 21983.3 at 20kN/m. For a span 8 m, at 3 kN/m, the cost/m³ is19195.5. Then increases to 20302.8 at7 kN/m and increases to 21353.8 at 10 kN/m. Then increases to 24632.21 at 20 kN/m.

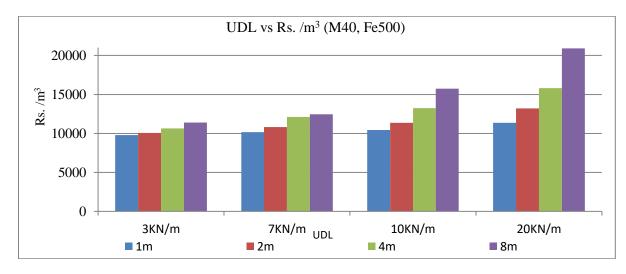


Figure 24. Variation of cost/m³ in Rs. with span for different loads (for Fe500, M40)

From figure 24, it can be observed that for a span 1 m, at 3 kN/m, the cost/m³ is 9791.277. It increases to 10157.06 at7 kN/m. Then it increases to 10431.41 at 10 kN/m and 11345.88 at 20 kN/m. For a span 2 m, at 3 kN/m, the cost/m³ is 10067.47. It increases to 10803.33 at 7 kN/m. Then it increases to 11355.22 at 10 kN/m and 13194.87 at 20 kN/m. For a span 4 m, at 3 kN/m, the cost/m³ is 10632.33. It increases to 12126.23 at 7 kN/m. Then it increases to 13246.3 at 10 kN/m and 15790.61 at 20kN/m. For a span 8 m, at 3 kN/m, the cost/m³ is 11375.87. Then increases to 12467.96 at 7 kN/m and increases to 15737.48 at 10 kN/m. Then increases to 20885.83 at 20 kN/m.

Part 4 presents the variation of $\cos t/m^3$ in Rs. for a given span and uniformly distributed load. From the results, it can be concluded that for lower grades of steel and concrete, the $\cos t/m^3$ in Rs. is almost the same for all spans and loads. At higher grades of concrete, it increases with an increase in span and load. However, the increase in $\cos t/m^3$ is significant when higher grades of steel are used than at lower grades of steel.

CONCLUSIONS

The following general conclusions can be drawn from this study.

- 1. Based on the study of variation of optimum percentage of steel for a given span and central point load/uniformly distributed load, it can be concluded that for lower grades of steel, the magnitude of the optimum percentage of steel is almost same for all spans at lesser loads while it increases with span for higher loads. However, for higher grades of steel, the variation of the percentage of steel with span and point load is not very significant.
- 2. Based on the study of variation of cost /m³ in Rs. for a given span and central point load, it can be concluded that for lower grades of steel and concrete, the cost/ m³ in Rs. is almost same for all spans and loads. At higher grades of concrete, it is constant for a given load and independent of the span. However, the cost /m³ increases with span for higher grades of concrete.
- 3. Based on the study of variation of cost /m³ in Rs. for a given span and uniformly distributed load, it can be concluded that for lower grades of steel and concrete, the cost/ m³ in Rs. is almost same for all spans and loads. At higher grades of concrete, it increases with an increase in span and load. However, the increase in cost /m³ is significant when higher grades of steel are used than at lower grades of steel.

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